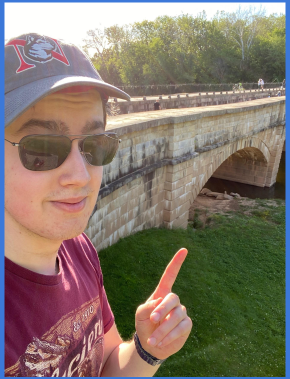
Music, Cards, and Game Shows!

Alex Roth, Grace Luft-Morgan, Ethan van Heerden, Emily Wang

Your Presenters

Alex:

- Grew up near DC
- Has played guitar for ten years
- Is planning to talk about the math in music!



Emily:

- From Northern NJ
- Plays tennis and dances
- Is planning to talk about the 21 Card Trick:)

Grace:

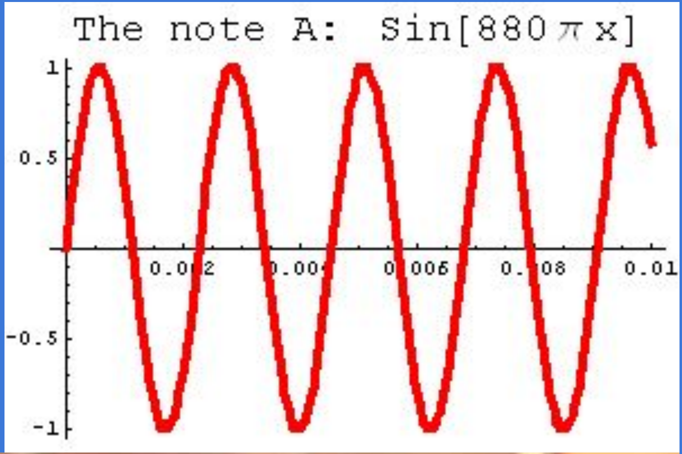
- Grew up in Westchester, NY
- Soccer and Horseback Riding
- Will discuss 27 and 8 Card Tricks

Ethan:

- From Northern NH
- Enjoys sports
- Has forklift licence
- Will talk about the Monty Hall problem

Math In Music

- Sound on a guitar is produced by plucking a stretched string, causing it to shoot back and forth quickly (oscillate)
- That make a soundwave with a certain frequency, which affects the pitch
- The string also vibrates in halves, thirds, fourths, fifths, etc, creating extra tones at octaves above the original, that persist even when the original stops - Overtones
- Everything in the universe has a resonant frequency, just like a guitar string
- Beatles Lost Chord - An extra note in a recording made it impossible for guitar players to replicate the chord, and no one knew why.




21 Card Trick

Instructions!

- Cards are dealt face-up into 3 columns of 7 cards each
- Player chooses one card and indicates its column
- Pick up the cards and deal them into 3 columns of 7 again
- Ask player which column the card is in
- Repeat this process once more
- Gather the cards and find the player’s card

How to do the Trick

- When picking up the columns, place the column with the player’s card in the middle of the three columns
- Repeat the process two more times
- The player’s card should be the 11th card from the top
- Can have variations when you place the column on the bottom or the top the last time and count the cards from there

How Does the Trick Work?

- The 21 card trick utilizes a process that “puts” the selected card in the middle of the deck and keeps it there using a pseudo-fixed point, which is the 11th card
- A similar trick can be devised for any odd number where the selected card is put in the middle

27 Card Trick

Instructions!

- Tell the player to pick a card in their head
- Deal the cards face up into three piles of 9
  - dealing left to right until each pile has 9 cards
- Ask the player to point to the pile that contains their card
- Place this pile on the bottom of the other two piles
- Deal the cards into 3 piles once again
- This time placing the pile pointed to by the player on top of the other two piles
- Deal the cards into 3 piles for a final time
- Place the pile pointed to by the player in the middle of the other two piles
- Your card will be revealed in space 12 of the deck

How Does the Trick Work?

GOAL: the players card to be in space 12

Ternary - using combinations of base 3 to represent a number

$$12-1 = 11 = (1x3^2)+(0x3^1)+(2x3^0)$$

$$(1x9)+(0x3)+(2x1)$$

$$(9)+(0)+(2)$$

$$11$$

“Coefficient Code”

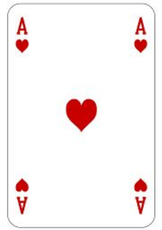
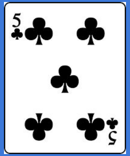
0: place the pile on the top
1: place the pile in the middle
2: place the pile on the bottom

$$12-1 = 11 = (\mathbf{1}x3^{2})+(\mathbf{0}x3^{1})+(\mathbf{2}x3^{0})$$

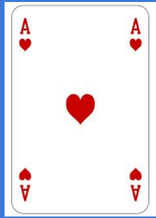
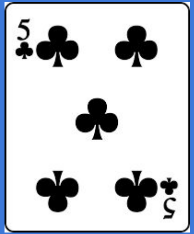
middle top bottom

RESULT: the players card to be in space 12




8 Card Trick

Instructions!

- Tell the player to pick a card in their head
- Deal the cards face up into two piles of 4
  - dealing left to right until each pile has 4 cards
- Ask the player to point to the pile that contains their card
- Place this pile on the bottom of the other pile
- Deal the cards into 2 piles once again
- This time placing the pile pointed to by the player on top of the other pile
- Deal the cards into 2 piles for a final time
- Place the pile pointed to by the player on the bottom of the other pile
- Your card will be revealed in space 6 of the deck

How Does the Trick Work?

<u>GOAL:</u> the players card to be in space 6

Binary - using combinations of base 2 to represent a number

$$6-1=5 = (1x2^2)+(0x2^1)+(1x2^0)$$

$$(1x4)+(0x2)+(1x1)$$

$$(4)+(0)+(1)$$

$$5$$

“Coefficient Code”

0: place the pile on the top
1: place the pile in the bottom

$6-1=5 = (\mathbf{1}x2^2)+(\mathbf{0}x2^1)+(\mathbf{1}x2^0)$

bottom top bottom

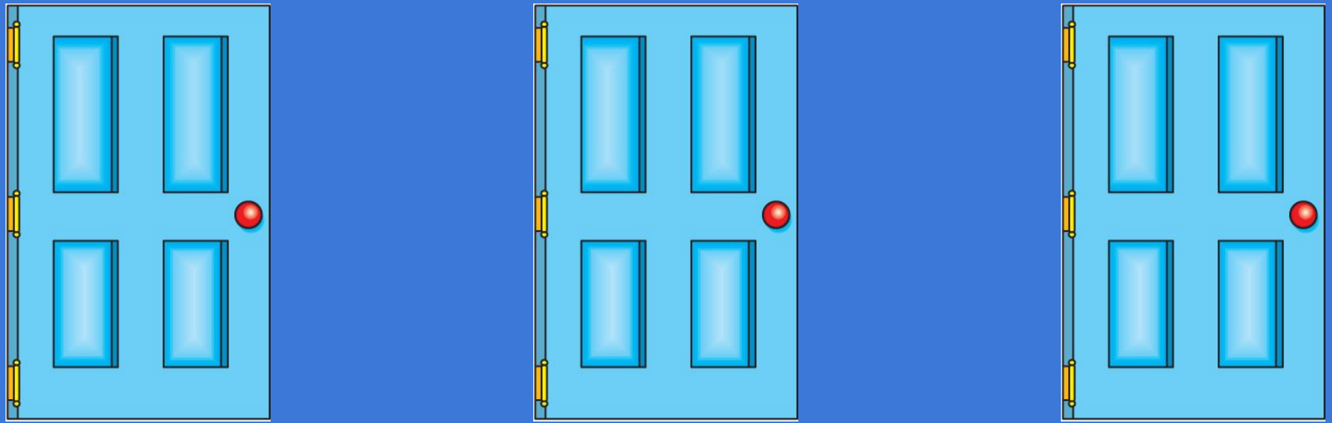
RESULT: the players card to be in space 6

The Monty Hall Problem

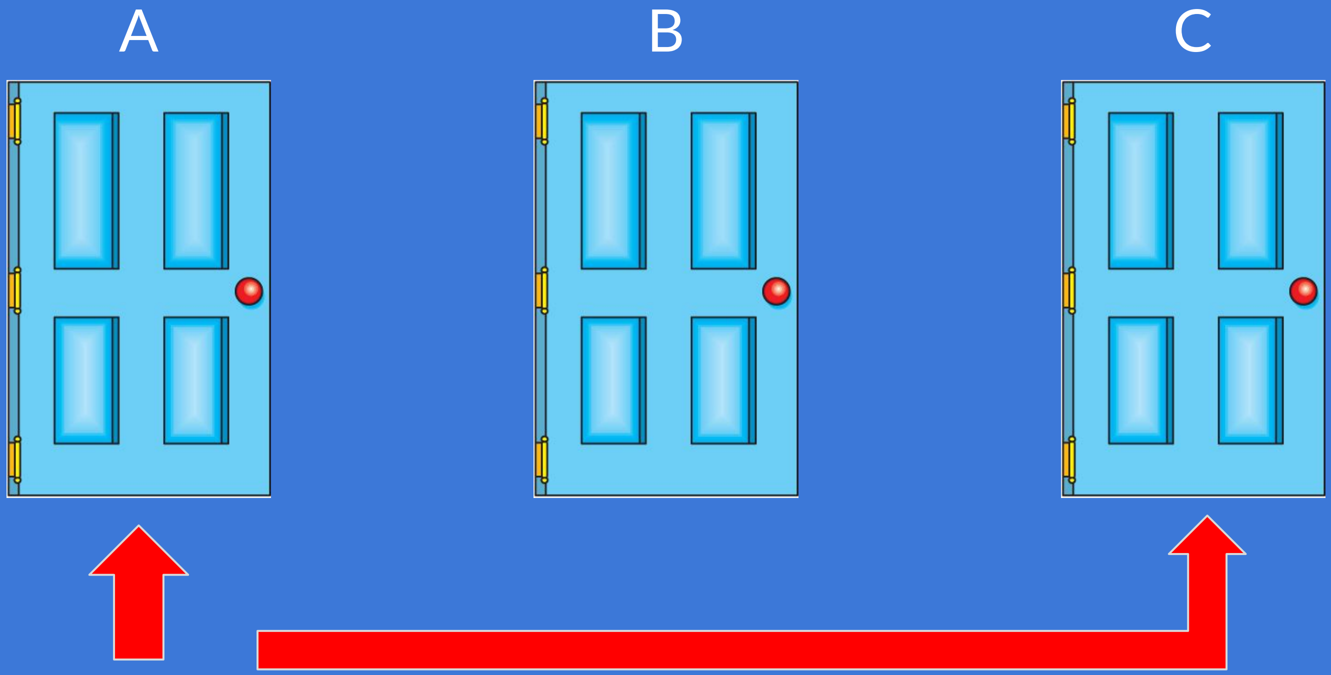
Ethan van Heerden

Background

- Comes from 1960s game show
  - Let’s Make a Deal
  - Hosted by **Monty Hall**
- There are 3 doors:



Should you switch doors?



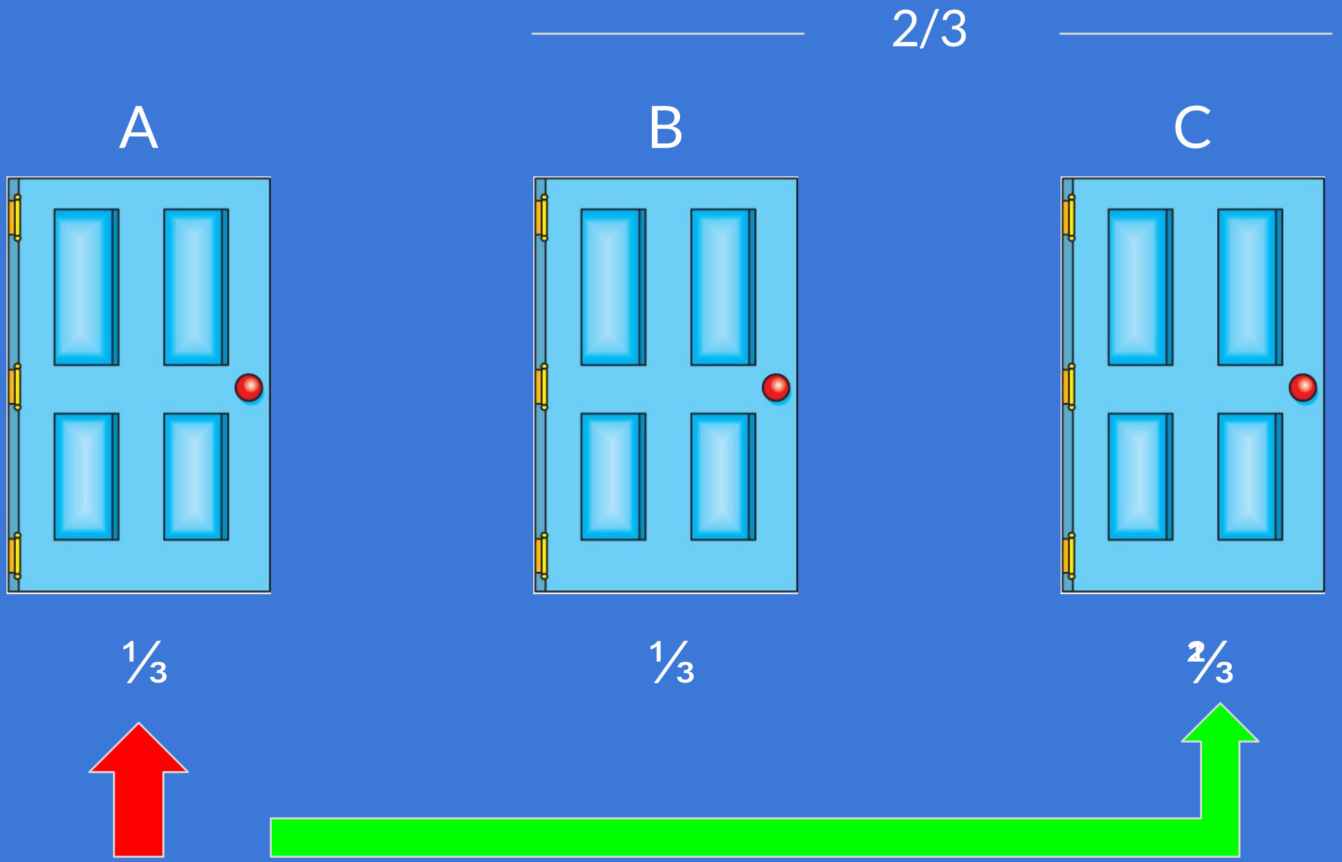
Let’s Play!

https://www.mathwarehouse.com/monty-hall-simulation-online/

Yes! You *always* should.

- A common misconception is that removing one of the doors with a goat makes the two doors equally likely to have the car. So it wouldn’t matter if you switched.

- But in reality it does matter!


2/3
A
B
C
1⁄3
1⁄3
2⁄3

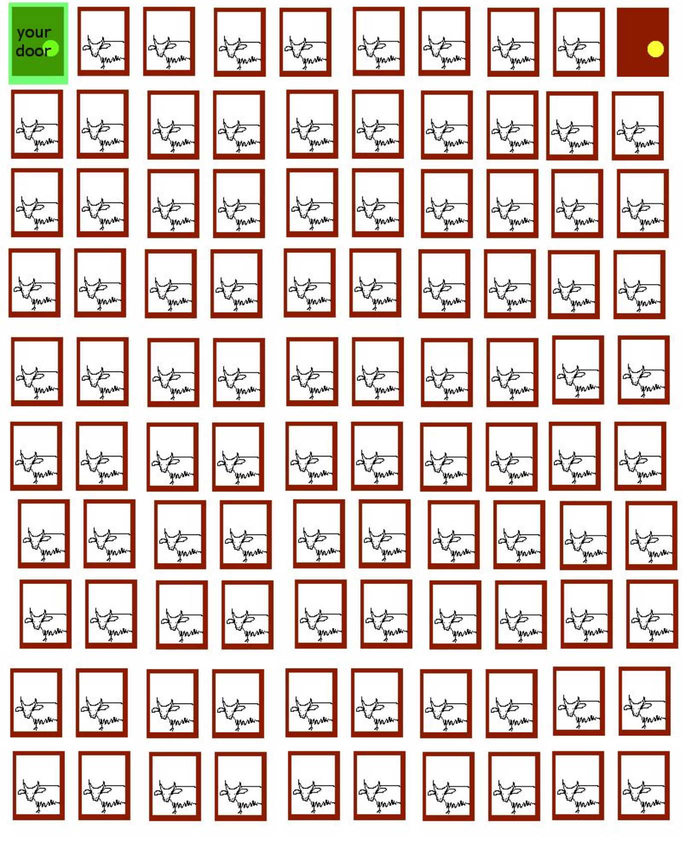
Conclusion, you should switch!

- By switching, we will double our chances of picking the door with the car. This can be difficult to understand, but here is the proof:
- https://www.mathwarehouse.com/monty-hall-simulation-online/

- What if we had 100 doors?

100 doors should help convince you

- You pick the first door. Chances are you most likely picked the wrong one.
- By picking the first door, there was a 99% chance the other doors had the car. So there is a 99% you'll win the car if you switch!



Still Don’t Believe Me?

Mathematical Proof with Bayes’ Theorem

Bayes’ Theorem is used to calculate the probability that an event occurs given additional information:

Bayes' Theorem

H: Hypothesis E: Observation

$$P(H|E) = \frac{P(E|H) * P(H)}{P(E)}$$

Say you pick door A and the host opens door C. We want to find the probability the car is behind door B:

$$P(Car\ behind\ B \mid Host\ opens\ C) = \frac{P(Host\ opens\ C \mid Car\ behind\ B) * P(Car\ behind\ B)}{P(Host\ opens\ C)}$$

$$P(Car\ behind\ B \mid Host\ opens\ C) = \frac{1 * \frac{1}{3}}{\frac{1}{2}}$$

$$P(Car\ behind\ B \mid Host\ opens\ C) = \frac{2}{3}$$

Thank you everyone!
Any Questions... about anything?